Modeling of Driver's Collision Avoidance Maneuver Based on Controller Switching Model

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***Abstract*—This paper presents a modeling strategy of human driving behavior based on the controller switching model focusing on the driver's collision avoidance maneuver. The driving data are collected by using the three-dimensional (3-D) driving simulator based on the CAVE Automatic Virtual Environment (CAVE), which provides stereoscopic immersive virtual environment. In our modeling, the control scenario of the human driver, that is, the mapping from the driver's sensory information to the operation of the driver such as acceleration, braking, and steering, is expressed by Piecewise Polynomial (PWP) model. Since the PWP model includes both continuous behaviors given by polynomials and discrete logical conditions, it can be regarded as a class of Hybrid Dynamical System (HDS). The identification problem for the PWP model is formulated as the Mixed Integer Linear Programming (MILP) by transforming the switching conditions into binary variables. From the obtained results, it is found that the driver appropriately switches the "control law" according to the sensory information. In addition, the driving characteristics of the beginner driver and the expert driver are compared and discussed. These results enable us to capture not only the physical meaning of the driving skill but the decision-making aspect (switching conditions) in the driver's collision avoidance maneuver as well.**

***Index Terms*—CAVE, collision avoidance, hybrid dynamical system, identification, MILP, PWP Model.**

I. Introduction

RECENTLY, the modeling of driving behavior has attracted great attention by many researchers [1]–[6], and this will also play an essential role in the design of a secure and safe vehicle operating system. Since the human driving behavior can be considered as a mapping from driver's sensory information to the operations of driver such as acceleration, braking, and steering, some linear controller models have been proposed [7]–[9]. Although the linear models enable us to capture the physical characteristics of the driving behavior intuitively, they sometimes mislead us to wrong understanding due to the high nonlinearity included in the human driving behavior. This tendency would be emphasized when the considered task becomes complex. In order to obtain more sophisticated driver model, some nonlinear dynamics based modeling is promising [10]. From this viewpoint, the Hidden Markov Models (HMMs), the nonlinear regression models, the neural networks and the fuzzy systems have been used [12]–[16]. These techniques, however, have some problems as follows: 1) The obtained model often results in too complicated model; 2) this makes it impossible to understand the physical meaning of the driving behavior; 3) the usefulness of information obtained by these models also remains questionable, especially for the design of driving assist system based on the driver model.

When we look at the driving behavior, it is often found that the driver appropriately switches the simple control laws instead of adopting the complex nonlinear control law. The switching mechanism can be regarded as a kind of driver's decision-making in the driving behavior. Therefore, it is highly recommended that the model of the driving behavior involve both physical skill (operation) and the decision-making aspect (switching condition). This kind of expression can be categorized into a class of Hybrid Dynamical System (HDS). HDSs are systems that consist of both continuous dynamics and logical conditions. The former are typically associated with the differential (or difference) equations and the latter with combinatorial logics, automata, and so on. Although many literatures have dealt with the expression, stability analysis, control, verification, and identification of the HDS in the control and computer science communities [11], the application of the HDS model to the analysis of the human behavior has not been discussed, as far as the authors know.

In this paper, the Piecewise Polynomial (PWP) model, which is a class of the HDS, is adopted to understand the human driving behavior especially focusing on the driver's collision avoidance maneuver. The driving data are collected by using the three-dimensional (3-D) Driving Simulator (DS) based on the CAVE Automatic Virtual Environment (CAVE), which provides stereoscopic immersive vision. Although our DS does not have any motion generator, thanks to the effect of the stereoscopic immersive vision, the examinee can feel the pseudo-acceleration. The advantages of using DS are 1) the safety of the examinee is always guaranteed, and 2) all environmental information can be captured without installing any sensor. In our modeling, the mapping from the driver's sensory information such as the range between cars, etc., to the operation of the driver such as steering, etc., are expressed by the PWP model. Then, we formulate the identification problem

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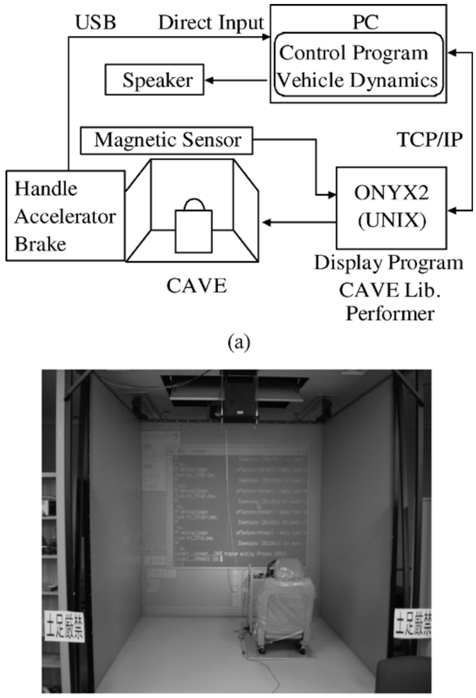


(b)

Fig. 1. Developed driving simulator. (a) Configuration of the DS. (b) CAVE system.

for the PWP model as the Mixed Integer Linear Programming (MILP) by transforming the logical conditions into inequalities [17]–[19]. By applying the developed modeling strategy, it becomes possible to discover not only coefficients in the polynomials but also parameters in the switching conditions from the measured driving data. This implies that both physical meaning of the driving skill and the decision-making aspect (switching condition) in the driving behavior can be identified simultaneously.

This paper is organized as follows. In Section II, configuration of the developed DS based on the CAVE is introduced. In Section III, the scenario of our examination is described. Based on the setup described in Section III, three drivers collision avoidance maneuvers are investigated in Section IV. In Section V, the modeling of the collision avoidance maneuver based on the expression as the PWP model is introduced, and identification results based on the MILP are shown.

II. Configuration of Driving Simulator

The configuration and appearance of the developed DS are shown in Fig. 1(a) and (b). The display unit in the CAVE system provides the stereoscopic immersive virtual environment, and it is controlled by ONYX2 (ONYX2-IR2 Desk Side). The display program was developed by making use of the CAVE library and the Performer. The cockpit is built by installing a real steering wheel, an accelerator, and a brake in the CAVE system. The information on the driver's operations using the steering wheel, accelerator, and brake are transferred to the PC through the USB terminal, and the vehicle position and orientation are calculated based on these amounts and vehicle dynamics implemented on the PC using the CarSim software. The results of the calculation are transferred to ONYX2 through the Internet (TCP/IP), and the 3-D visual image based on the position and orientation of the vehicle shows up on the screen.

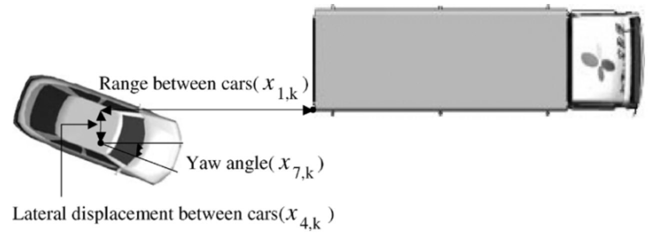


Fig. 2. Definition of physical variables.

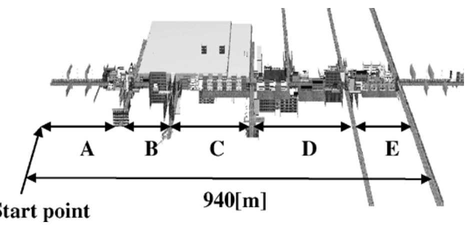


Fig. 3. Road environment for experiments.

III. Data Acquisition in Collision Avoidance

A. Experimental Environment and Conditions

In this paper, we focus on the driver's collision avoidance maneuver at the instance of the sudden stopping of the preceding vehicle.

In order to model the driver's collision avoidance maneuver, the following sensory information is captured as the inputs to the driver:

1) range between cars ($x_{1,k}$);
2) range rate (time derivative of $x_{1,k}$: $x_{2,k}$);
3) acceleration of examinee's car ($x_{3,k}$);
4) lateral displacement between cars ($x_{4,k}$);
5) lateral relative velocity (time derivative of $x_{4,k}$: $x_{5,k}$);
6) lateral relative acceleration (time derivative of $x_{5,k}$: $x_{6,k}$);
7) yaw angle ($x_{7,k}$);
8) yaw rate (time derivative of $x_{7,k}$: $x_{8,k}$);
9) yaw acceleration (time derivative of $x_{8,k}$: $x_{9,k}$).

The definition of variables $x_{1,k}$, $x_{4,k}$, and $x_{7,k}$ are illustrated in Fig. 2.

The outputs of drivers are also specified as follows:

1) braking amount ($y_{1,k}$);
2) steering amount ($y_{2,k}$).

In these definitions, $k \in Z$ denotes a discrete sampling index. Note that no acceleration operation is necessary in the collision avoidance maneuver.

The configuration of the virtual environment developed for the experiment is shown in Fig. 3. It has four intersections and two T-type junctions. The road is 940 m long and 7 m wide, and the pedestrian way is 1.5 m wide. The friction coefficient of the road was set to be 0.8. The scene around the start point is shown in Fig. 4.

The vehicle moves from the left side to the right side in Fig. 3. After the 940-m point, the same environment from the start point

Fig. 4. Road image at start point.

shows up again. Therefore, the driver feels that this virtual environment is straight road with no end point. There exist only two vehicles in this environment. One of them is a sedan-type car driven by the examinee. The other one is the big truck, which runs in front of the examinee and is controlled by the operator of the experiment. The examinee's car used in the simulator has an engine with 3000-cc displacement. In addition, the car is supposed to have an antilock braking system and a range keep control, in which the distance between cars is kept within certain range by the controller. The truck in front of the examinee runs at the constant speed of 50 km/h. The maximum deceleration of the truck is supposed to be 7 m/s^2.

B. Procedure of Experiment

The examinee drives the car keeping constant range (26 m) to the preceding truck. Quantitatively speaking, this range was decided by considering the parameter called Time To Collision (TTC), which is defined by

$$\text{TTC} = \frac{\text{range } (x_{1,k})}{\text{velocity of vehicle}}. \quad (1)$$

In our case, TTC was about 2 s, and this value is generally recognized as "sufficient range" to avoid the collision.

The operator set the red or green parking vehicles on the right side at each intersection. The examinee is supposed to take a look at the right side at each intersection, and then, the examinee answers the color of the parking vehicle. When the examinee looks right side at some intersection (randomly chosen), the preceding truck is supposed to stop with maximum deceleration. Then, the collision avoidance maneuver of the examinee is measured. By adopting this examination procedure, we can exclude the "effect of prediction" of the examinee, which comes from the iterative experiments. Four trials have been made by each examinee. Before doing the experiments, all examinee practiced avoiding the stopped truck to get used to the driving simulator in advance. Then, each driver took about 10 min to complete all of the trials.

IV. Measured Data and Modeling Framework

Based on the setup described in Section III, three drivers carried out the avoiding tasks under virtual environments. The collision avoidance maneuver is characterized by the profile between the beginning and ending of the steering operation. The personal information of all three examinees are listed in Table I.

TABLE I
Individual Information of Examinees

Examinee	Age [Years old]	Mileage per year [km]	Driving Career [Years]
E1	23	1000	1
E2	23	4000	4
E3	29	15000	10

Six profiles of the driving data (two trials from each driver) are depicted in Figs. 5–10. In these figures, the Ei-j denotes the jth trial data of the ith driver. In all figures, (a) illustrates the overview of the collision avoidance maneuver. The mark in (a) designated by $*$ represents the location of the examinee's vehicle when the preceding truck stops. In the figures, (b) shows the trajectories in $x_1 - y_1, y_2$ (range – braking, steering) spaces. In (c) the time profiles of the steering and braking are shown, and (d) shows the time profiles of the range, range rate, and lateral displacement.

Actually, we have asked the examinees to avoid the preceding vehicle by emphasizing the steering operation rather than the braking operation. Due to this instruction, in the measured data, the E2 and E3 did not use braking at all (see Figs. 7–10). On the other hand, since the E1 was a beginner driver, he could not avoid the collision only with steering. As a result, he used braking to secure safety. From these observations, the role of braking in our setup is considered to "assist" the avoidance task with steering. Therefore, the steering operation seems to include higher level decision making than the braking operation. In the following analysis, we focus only on the relationship between the sensory information and the steering operation.

Fig. 11 also shows the projected images of three driving scenes of the E2-2 designated by (1), (2), and (3) in Fig. 8(a). When we look at trajectories in the (a) part of the figures in all trials, roughly speaking, the collision avoidance maneuver can be regarded as the series of the following four sub-maneuvers: 1) the first period of avoidance; (2) the second period of avoidance; (3) the first period of recovery; and (4) the second period of recovery. In the following, we denote these four periods "Mode A," "Mode B," "Mode C," and "Mode D." The more formal algebraic expression of this maneuver based on the PWP model is described as follows:

1) *Mode A (first period of avoidance)*

$$y_{2,k} = a_0 x_{1,k} + a_1 x_{2,k} + a_2 x_{4,k}, \qquad \text{if } x_{1,k} > e_1. \quad (2)$$

2) *Mode B (second period of avoidance)*

$$y_{2,k} = b_0 x_{1,k} + b_1 x_{2,k} + b_2 x_{4,k}, \qquad \text{if } e_1 \geq x_{1,k} > e_2. \quad (3)$$

3) *Mode C (first period of recovery)*

$$y_{2,k} = c_0 x_{1,k} + c_1 x_{2,k} + c_2 x_{4,k}, \qquad \text{if } e_2 \geq x_{1,k} > e_3. \quad (4)$$

4) *Mode D (second period of recovery)*

$$y_{2,k} = d_0 x_{1,k} + d_1 x_{2,k} + d_2 x_{4,k}, \qquad \text{if } e_3 \geq x_{1,k}. \quad (5)$$

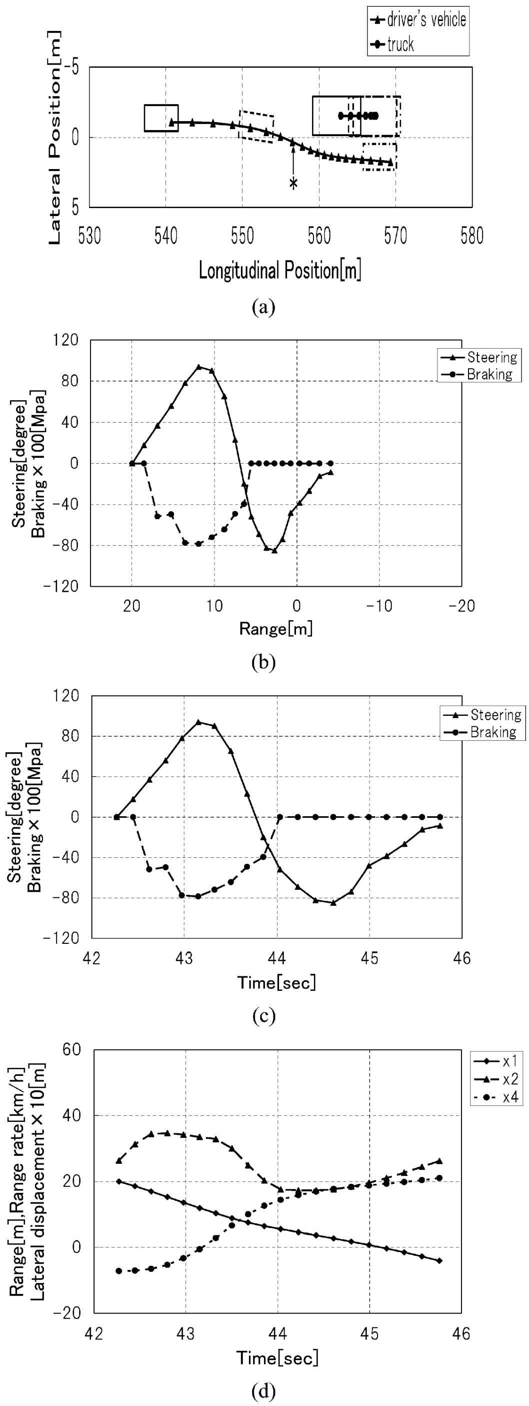


Fig. 5. Profiles of the avoiding task of E1-1.

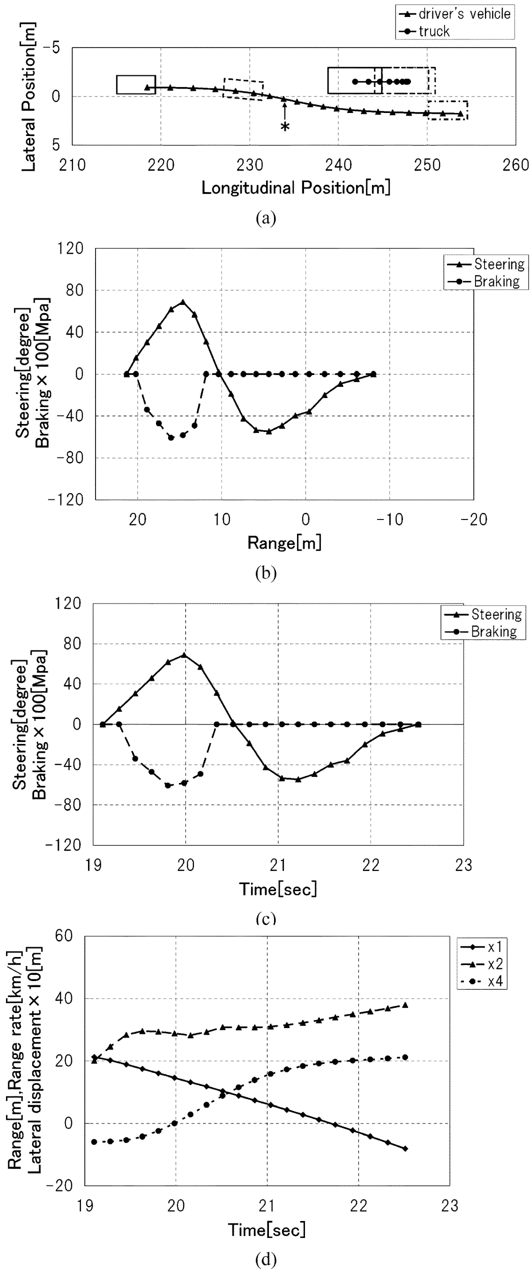


Fig. 6. Profiles of the avoiding task of E1-2.

For this model, we give some discussions in the following: First of all, although the above model does not include dynamics (i.e., just static mapping only), it is straightforward to extend it to the model including dynamics by adopting AutoRegressive (AR) model. Second, only $x_{1,k}$, $x_{2,k}$, and $x_{4,k}$ among many sensory information appear in the right-hand side in (2) to (5). In our past study, we have clarified that $x_{1,k}$, $x_{2,k}$, and $x_{4,k}$ played important roles in the collision avoidance maneuver by applying the Group Method of Data Handling (GMDH) technique [2], [20]. Finally, one may argue that it is the combination of the range and range rate that affect on the switching of behavior. The starting point of the steering operation may strongly depend on the range rate. In this work, however, this point is out of our consideration. We have focused only on the behavior after the beginning of the steering operation. In this case, since the range rate does not vary so significantly (except the mode A) compared with the range between cars (it monotonically decreases as the time evolves), the switching mechanism can be expected to be described as the function of the range only.

The graphic representation of this behavior, which is based on the Hybrid Automaton (HA) expression, is depicted by Fig. 12. In (2) to (5), e_1, e_2, and e_3 are parameters to specify the switching condition between modes. These parameters can be regarded as parameters that are closely related to the decision-making aspect in the human driver. Although these

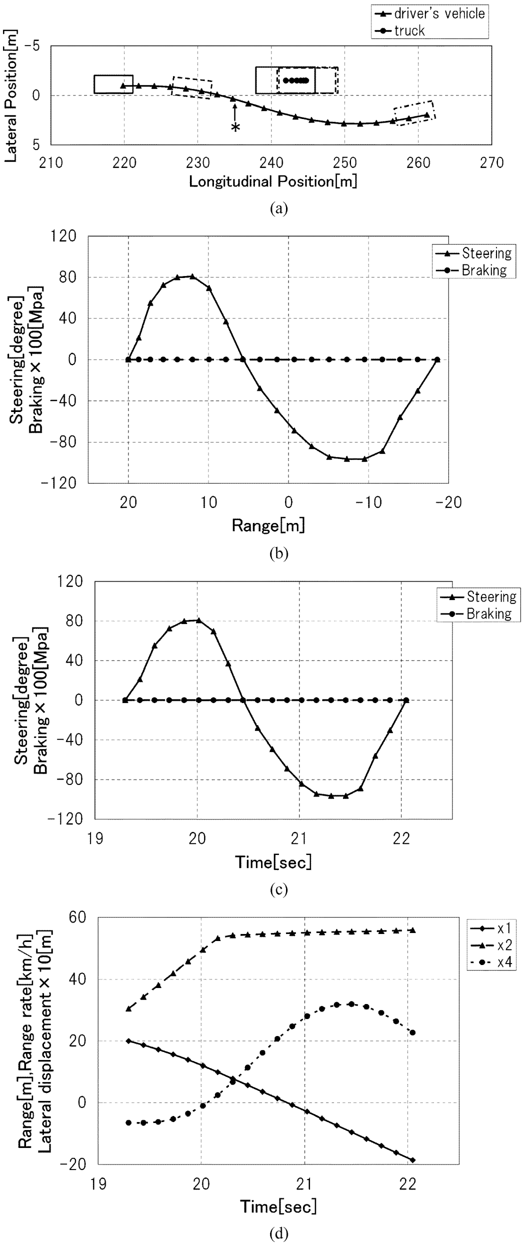


Fig. 7. Profiles of the avoiding task of E2-1.

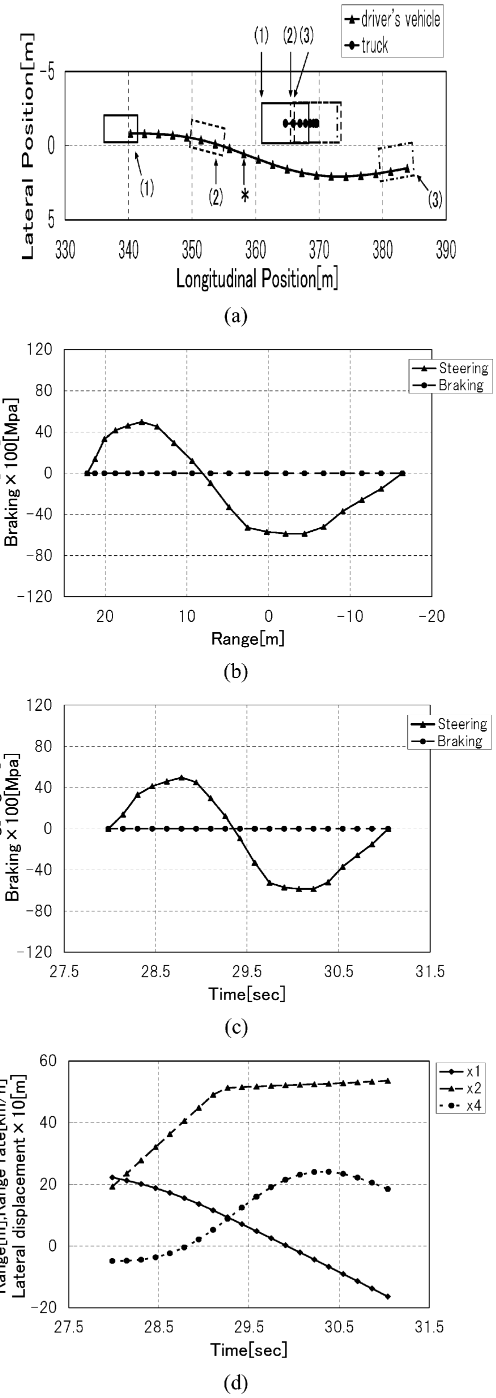


Fig. 8. Profiles of the avoiding task of E2-2.

parameters cannot be measured directly, they can be estimated from the measured data in our proposed technique introduced in the next section.

Since the structure of this model contains both the continuous dynamics (polynomials) and the logical conditions (switching of polynomials), the proposed model belongs to a kind of HDS. This kind of HDS model enables us to capture not only the physical meaning (polynomials) but the decision-making aspect (logical conditions) in the driving behavior as well. As mentioned above, since the switching conditions between each interval are not specified in advance in our problem setup, the parameters specifying switching conditions (e_1 to e_3) and coefficients appearing in each polynomial (a_i to d_i) must be discovered simultaneously from the measured data. In the next section, the strategy to solve this simultaneous identification problem is introduced.

V. Modeling of Driver's Behavior Based on PWP Model and MILP

A. *Useful Tools for Identification of PWP Model*

The goal of our modeling is to discover not only coefficients in the polynomials a_i, b_i, c_i, and d_i but also parameters in the "switching conditions" e_i from the measured driving data.

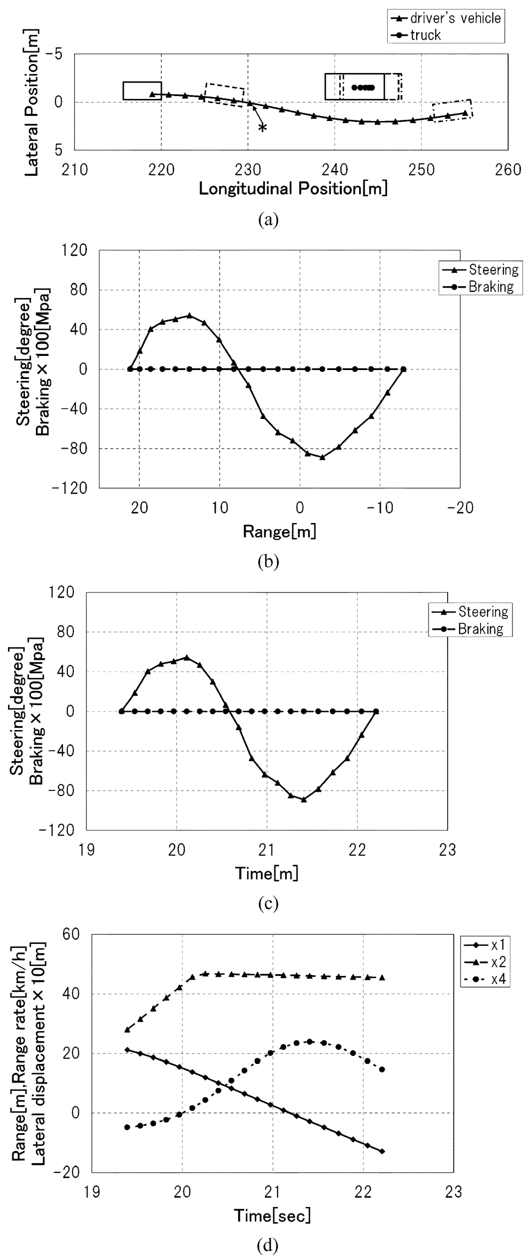


Fig. 9. Profiles of the avoiding task of E3-1.

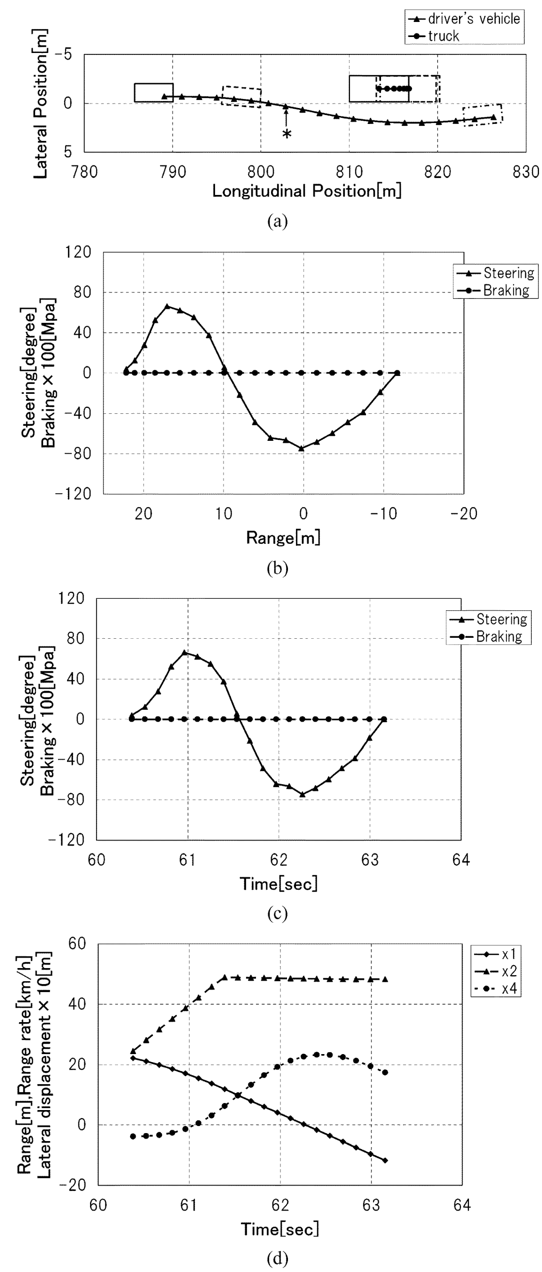


Fig. 10. Profiles of the avoiding task of E3-2.

Although this identification problem is not straightforward to handle, the idea developed in the Mixed Logical Dynamical Systems framework [18], [19] makes it tractable. The key idea is to transform the logical condition into some linear inequalities by introducing auxiliary binary variables $\delta \in \{0, 1\}$ and auxiliary continuous variables z and to formulate the problem as the MILP.

In the following, some useful tools to transform the logical condition into linear inequalities are introduced. First, the logical relationship given by

$$[f(x) \geq a] \Leftrightarrow [\delta = 1] \tag{6}$$

can be transformed into the inequalities

$$-f(x) + (a - m)\delta + m \leq 0 \tag{7}$$
$$f(x) - (M - a + \epsilon)\delta - a + \epsilon \leq 0 \tag{8}$$

where $M = \max_x f(x)$, $m = \min_x f(x)$, and $\epsilon > 0$ is a small tolerance. In addition, in our setup, the product term of binary and continuous variables such as $\delta f(x)$ often appear. Since it is undesirable to handle this nonlinear term, we next introduce another auxiliary variable $z = \delta f(x)$, which satisfies the following two logical relationships:

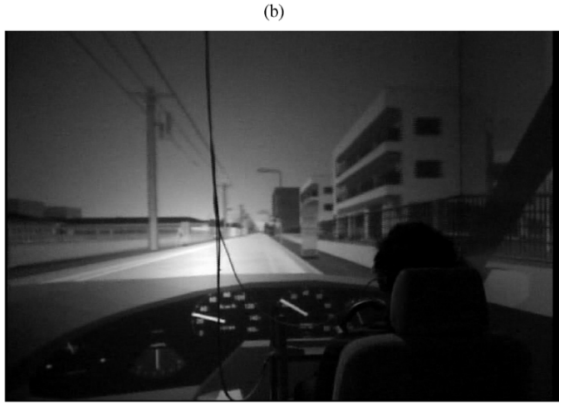
$$[\delta = 0] \Rightarrow [z = 0], \; [\delta = 1] \Rightarrow [z = f(x)]. \tag{9}$$

These relationships can be transformed into the following equivalent inequalities:

$$z \leq M\delta \tag{10}$$
$$-z \leq -m\delta \tag{11}$$
$$z \leq f(x) - m(1 - \delta) \tag{12}$$
$$-z \leq -f(x) + M(1 - \delta). \tag{13}$$

(a)

(b)



(c)

Fig. 11. Projected image of three driving scenes of E2-2. (a) Scene at (1). (b) Scene at (2). (c) Scene at (3).

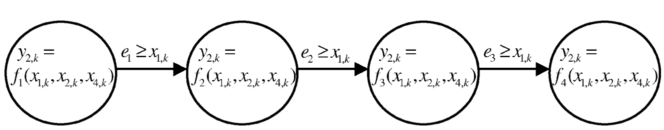


Fig. 12. Graphical representation of PWP model.

B. Identification of PWPS Model by MILP

In order to transform the four logical conditions involved in (2) to (5) into the equivalent inequalities, binary variables $\delta_{1,k}$, $\delta_{2,k}$, and $\delta_{3,k}$ are introduced as follows:

1) $[x_{1,k} > e_1] \Leftrightarrow [\delta_{1,k} = 0, \delta_{2,k} = 0, \delta_{3,k} = 0]$.
2) $[e_1 \geq x_{1,k} > e_2] \Leftrightarrow [\delta_{1,k} = 1, \delta_{2,k} = 0, \delta_{3,k} = 0]$.
3) $[e_2 \geq x_{1,k} > e_3] \Leftrightarrow [\delta_{1,k} = 0, \delta_{2,k} = 1, \delta_{3,k} = 0]$.
4) $[e_3 \geq x_{1,k}] \Leftrightarrow [\delta_{1,k} = 1, \delta_{2,k} = 1, \delta_{3,k} = 1]$.

By applying the transformation rules and introducing the auxiliary variables, (2) to (5) can be rewritten as the following equation:

$$\begin{aligned} y_{2,k} = {} & a_0 x_{1,k} + a_1 x_{2,k} + a_2 x_{4,k} + x_{1,k} z_{1,k} \\ & + x_{2,k} z_{2,k} + x_{4,k} z_{3,k} + x_{1,k} z_{4,k} + x_{2,k} z_{5,k} \\ & + x_{4,k} z_{6,k} + x_{1,k} z_{7,k} + x_{2,k} z_{8,k} + x_{4,k} z_{9,k} \end{aligned} \tag{14}$$

where the auxiliary variables $z_{i,k}$ are defined as follows:

$$z_{i,k} = \delta_{1,k}(b_{i-1} - a_{i-1}) \quad (i = 1 \sim 3) \tag{15}$$
$$z_{i,k} = \delta_{2,k}(c_{i-4} - a_{i-4}) \quad (i = 4 \sim 6) \tag{16}$$
$$z_{i,k} = \delta_{3,k}(a_{i-7} - b_{i-7} - c_{i-7} + d_{i-7}) \quad (i = 7 \sim 9) \tag{17}$$
$$z_{i,k} = \delta_{1,k} e_{i-9} \quad (i = 10 \sim 11) \tag{18}$$
$$z_{i,k} = \delta_{2,k} e_{i-11} \quad (i = 12 \sim 14) \tag{19}$$
$$z_{i,k} = \delta_{3,k} e_{i-14} \quad (i = 15 \sim 17). \tag{20}$$

In addition, as stated in the previous section, some linear inequalities that come up with the introduction of $\delta_{i,k}$ and $z_{i,k}$ must be accompanied with (14) to (20).

Now, the problem to find the coefficients in the polynomials and the parameters in the switching condition is formulated as the following MILP:

$$\begin{aligned} & \text{known } y_{2,k}, x_{1,k}, x_{2,k}, x_{4,k} \\ & \text{find } \{a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2 \\ & d_0, d_1, d_2, e_1, e_2, e_3, \delta_{1,k}, \delta_{2,k}, \delta_{3,k}\} \\ & \text{which minimize} \quad J = \sum_{k=1}^{N} |y_{2,k} - \hat{y}_{2,k}| \end{aligned} \tag{21}$$

subject to

$$z_{i,k} \leq M_i \delta_{j,k} \tag{22}$$
$$-z_{i,k} \leq -m_i \delta_{j,k} \tag{23}$$
$$z_{i,k} \leq f - (1 - \delta_{j,k}) m_i \tag{24}$$
$$-z_{i,k} \leq -f + (1 - \delta_{j,k}) M_i \tag{25}$$
$$\delta_{1,k} \in \{0,1\},\ \delta_{2,k} \in \{0,1\},\ \delta_{3,k} \in \{0,1\} \tag{26}$$
$$0 \leq \delta_{1,k} + \delta_{2,k} - \delta_{3,k} \leq 1 \tag{27}$$
$$0 \leq \delta_{1,k} - \delta_{3,k} \leq 1,\ 0 \leq \delta_{2,k} - \delta_{3,k} \leq 1 \tag{28}$$
$$\delta_{2,k} \leq \delta_{2,k+1},\ \delta_{3,k} \leq \delta_{3,k+1} \tag{29}$$
$$\begin{aligned} x_{1,k} \leq {} & (1 - (\delta_{1,k} + \delta_{2,k}) + \delta_{3,k}) M_e + z_{10,k} + z_{13,k} \\ & - z_{15,k} - z_{16,k} + z_{17,k} \end{aligned} \tag{30}$$
$$\begin{aligned} x_{1,k} \geq {} & e_1 - z_{10,k} + z_{11,k} - z_{12,k} + z_{14,k} + z_{15,k} \\ & - z_{16,k} - z_{17,k} + \epsilon + \delta_{3,k}(m_e - \epsilon) \end{aligned} \tag{31}$$
$$\begin{aligned} \hat{y}_{2,k} = {} & a_0 x_{1,k} + a_1 x_{2,k} + a_2 x_{4,k} + x_{1,k} z_{1,k} \\ & + x_{2,k} z_{2,k} + x_{4,k} z_{3,k} + x_{1,k} z_{4,k} + x_{2,k} z_{5,k} \\ & + x_{4,k} z_{6,k} + x_{1,k} z_{7,k} + x_{2,k} z_{8,k} + x_{4,k} z_{9,k} \end{aligned} \tag{32}$$

where f is substituted for the continuous part of $z_{i,k}$ (i.e., $b_0 - a_0$, e_1, etc.). M_i and $m_i (i = 1 \sim 17)$ are the maximum and minimum values of $z_{i,k}$, and M_e and m_e represent the maximum and minimum values of e_1 to e_3, respectively. ϵ is a small tolerance.

Here, we have used the performance index $\mathbf{J}$ in (21) and not the root mean square errors. The reason is to formulate the problem as an MILP. If we adopt root mean square errors, the identification problem leads to Mixed Integer Quadratic Programming (MIQP). MILP has a much bigger computational advantage compared with MIQP.

There are several ways to solve the MILP. One of the most efficient algorithms is a branch-and-bound method. Although it requires some heuristic rules in the decision of the branching and bounding operation, it can guarantee the global optimality and can reduce the computational burden with the assistance of appropriate heuristic rules.

Note that the computational burden strongly depends on the number of binary variables since this specifies the size of the search space. In our case, the number of the sample points affects the computational burden significantly.

Finally, we address the following two important problems to extend the model: First, as described in Section IV, if we try to include the range rate in the switching condition like

$$x_1 + e_1 x_2 \geq e_2$$

one more trial data (totally two trial data; this is minimum requirement) must be input to the identification procedure. (Otherwise, the parameters e_1 and e_2 in the switching condition shown in the above inequality cannot be determined uniquely.) Although this case may be tractable theoretically, unfortunately, unreasonably large computational effort is required in our current framework. The more complicated switching mechanism will be addressed in our future work.

Second, generally speaking, the braking and steering operations may interact with each other. The difference of the amount of the braking certainly leads to the different steering operation. Moreover, for the more complicated task, the steering and braking operations may interact with in more complicated manner. In this case, analyses of both the steering and braking will be required. Development of the driver's model including both the braking and steering operations can be done by adopting either of following two scenarios.

1) The steering and braking operations are modeled independently by regarding them as outputs from two single-output controllers. In this case, we will see different switching scenarios for steering and braking. Therefore, some coordination model to integrate them must be developed.
2) The steering and braking operations are modeled simultaneously by regarding them as outputs from one multi-output controller. Even in this case, a developed identification algorithm will return satisfactory results. However, it may not be straightforward to give a clear interpretation for the obtained switching points since the time intervals where steering and braking are active are different

C. Identification Results and Discussions

Based on the formulation of the identification of the PWP model described in the previous subsection, the identification of coefficients in the polynomials and parameters in the switching conditions was carried out. The 20 sampling data of the six trials shown in the previous section were used for the identification. These data were selected by culling from the measured data with longer sampling interval (around 150 ms, depending on the length of the profile). Before applying the MILP to the measured data, all input data were normalized as follows:

$$\bar{x}_i = \begin{cases} \frac{x_i}{|x_{i\,\max}|} & (|x_{i\,\max}| \geq |x_{i\,\min}|) \\ \frac{x_i}{|x_{i\,\min}|} & (|x_{i\,\max}| < |x_{i\,\min}|) \end{cases} \quad x_i \in [x_{i\,\min}\ x_{i\,\max}]. \tag{33}$$

Output data y_i were also normalized in the same way.

All numerical experiments for parameter identification have been performed by PC (CPU Pentium 4 3.06 GHz and Memory 1024 MB). We have used commercial software called NUOPT to solve MILP. It took about ten minutes to find the solution for each trial data. In order to verify the validity of the obtained PWP model, the reproduced steering profiles generated by the identified parameters are plotted together with the measured steering profiles and the identified switching points in Figs. 13–18.

In Figs. 13–18, the horizontal and vertical axes represent the range between cars and the steering amount, respectively. In the steering amount, the right and left turn take positive and negative values, respectively. In addition, the switching points between polynomials (e_i) are designated by vertical lines. As shown in Figs. 13–18, the measured and reproduced steering profiles agree well with each other. These results verify the validity of the modeling based on the PWP model.

In order to understand the characteristics in the collision avoidance maneuver, the identified parameters for these six trials are analyzed in the following. The identified coefficients in the polynomials a_i, b_i, c_i, and d_i, the parameters in the switching conditions e_i, and the value of objective function J are listed in Tables II–IV.

Based on these results, we discuss the collision avoidance maneuver in the following.

1) Coefficients in Polynomials (a_i, b_i, c_i, d_i): Although it is not straightforward to give rigorous interpretation of all identified coefficients listed in the tables, some interesting characteristics can be found. First, similar control coefficients are collected and classified in each mode by referring to the magnitude and sign of parameters as follows:

- Mode A: (E1-1, E1-2), (E2-1, E3-2), (E2-2, E3-1);
- Mode B: (E1-1, E1-2), (E2-1, E2-2, E3-1, E3-2);
- Mode C: (E1-1, E1-2, E3-1, E3-2), (E2-1, E2-2);
- Mode D: (E1-1, E1-2, E3-1, E3-2), (E2-1, E2-2).

The driving characteristics of each mode can be summarized as follows.

- *Mode A:* The magnitude of a_2 is greater than the magnitudes of a_0 and a_1 in all trials. Since the measured variables are normalized, this implies that all drivers signify the lateral displacement. In addition, $a_2 > 0$ holds for all six trials. On the other hand, although the driver E1 used similar control coefficients in two trials, the drivers E2 and E3 used different control coefficients in each trial (sign of a_0 and a_2 were changed). This implies that there exist two control ways to avoid the collision in mode A. The difference of coefficients between E2-1 and E2-2 (also E3-1 and E3-2) may have some relation to the difference of the

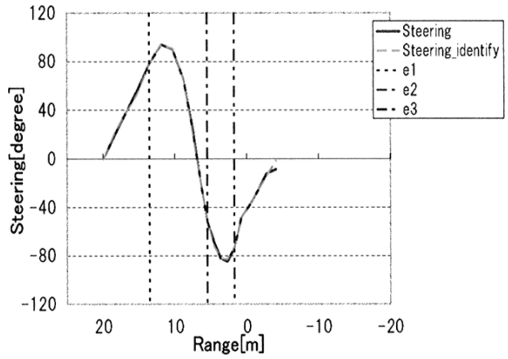


Fig. 13. Comparison of measured and reproduced steering profiles (E1-1).

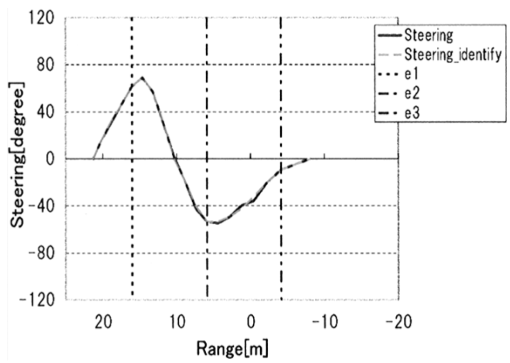


Fig. 14. Comparison of measured and reproduced steering profiles (E1-2).

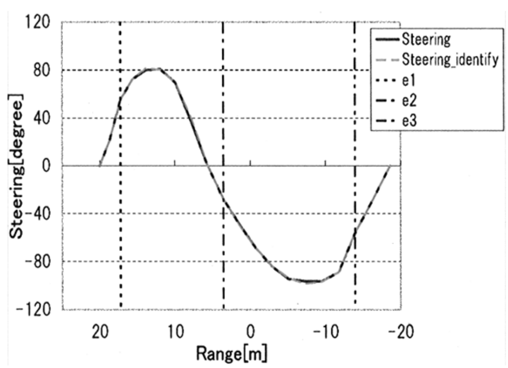


Fig. 15. Comparison of measured and reproduced steering profiles (E2-1).

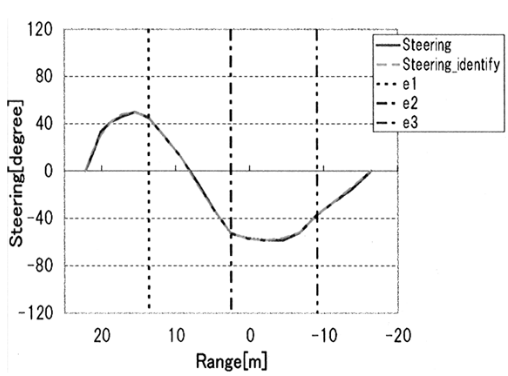


Fig. 16. Comparison of measured and reproduced steering profiles (E2-2).

switching point e_1. The e_1 in the E2-1 and E3-2 emerged "before the peak" of the steering profile, and the e_1 in the E2-2 and E3-1 emerged "after the peak" of it (see Fig. 15).

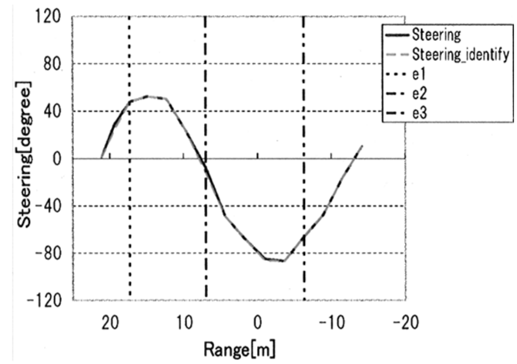


Fig. 17. Comparison of measured and reproduced steering profiles (E3-1).

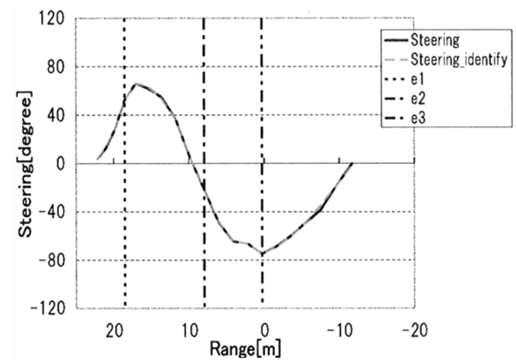


Fig. 18. Comparison of measured and reproduced steering profiles (E3-2).

TABLE II
IDENTIFIED PARAMETERS OF E1

E1-1		E1-2	
Parameters	Values	Parameters	Values
a_0	0.00693	a_0	-0.09937
a_1	1.30003	a_1	1.67221
a_2	2.90066	a_2	2.81377
b_0	-3.34984	b_0	6.56987
b_1	3.05732	b_1	-4.61618
b_2	-1.53960	b_2	1.33592
c_0	-1.82772	c_0	-2.06471
c_1	5.00378	c_1	2.01813
c_2	-3.76207	c_2	-2.48792
d_0	-0.98035	d_0	-0.70246
d_1	2.55111	d_1	0.58586
d_2	-2.14391	d_2	-0.85200
e_1	13.53680	e_1	16.02157
e_2	5.49659	e_2	5.89680
e_3	1.76500	e_3	-4.13509
J	21.01392	J	12.98787

- *Mode B:* The E2 and E3 used similar control coefficients, i.e., b_0, $b_2 < 0$, and $b_1 > 0$, and the magnitude of the b_2 is greater than the magnitude of b_0 and b_1. On the other hand, the E1 used quite different control parameters. The magnitude of b_0 is greater than the magnitude of b_1 and b_2. This implies that the E1 signifies the range in the mode B. This phenomenon can be explained by the use of braking of the E1 (recall that the E2 and E3 did not use the braking

TABLE III
IDENTIFIED PARAMETERS OF E2

E2-1		E2-2	
Parameters	Values	Parameters	Values
a_0	0.47690	a_0	-1.77492
a_1	3.88868	a_1	2.64713
a_2	12.86150	a_2	-4.05504
b_0	-1.94962	b_0	-1.22867
b_1	2.11491	b_1	2.06895
b_2	-4.18527	b_2	-3.45303
c_0	-0.26789	c_0	-0.81514
c_1	0.69743	c_1	0.48936
c_2	-1.82009	c_2	-1.61568
d_0	-1.66699	d_0	-1.38185
d_1	-0.84566	d_1	-0.34956
d_2	-0.98870	d_2	-0.87574
e_1	17.23320	e_1	13.62359
e_2	3.57729	e_2	2.52140
e_3	-13.93120	e_3	-9.09120
J	7.69846	J	11.94363

TABLE IV
IDENTIFIED PARAMETERS OF E3

E3-1		E3-2	
Parameters	Values	Parameters	Values
a_0	-2.56116	a_0	0.91158
a_1	2.57909	a_1	1.42069
a_2	-5.09277	a_2	9.56403
b_0	-3.21041	b_0	-1.50567
b_1	3.01934	b_1	2.30933
b_2	-3.70968	b_2	-3.71769
c_0	-2.36665	c_0	-6.70262
c_1	2.52844	c_1	5.98513
c_2	-3.47618	c_2	-6.74154
d_0	-0.45905	d_0	-1.29799
d_1	0.92433	d_1	0.23372
d_2	-1.92777	d_2	-1.24855
e_1	15.50928	e_1	18.53998
e_2	4.56570	e_2	7.97859
e_3	-0.96239	e_3	0.27150
J	15.70701	J	9.01106

TABLE V
AVERAGE AND VARIANCE OF SWITCHING POINTS

Parameter	Average[m]	Variance[m^2]
e_1	16.14841	5.03367
e_2	5.72993	4.69821
e_3	-4.68751	22.13705

at all). Since E1 is the beginner driver, the E1 relied on the range information rather than the lateral displacement. Thus, this mode clearly shows the difference between the beginner and the expert (in other words, the effect of using braking).

- *Mode C:* The sign of all coefficients are consistent in this mode. This mode shows highest similarity among four modes. The only difference seen in this mode was the magnitude of coefficients. The E2 used relatively small control coefficients.

TABLE VI
COMPARISON OF J

Trial data	4 mode	3 mode
E1-1	21.01392	57.76640
E1-2	12.98787	30.46904
E2-1	7.69846	17.85463
E2-2	11.94363	20.31397
E3-1	15.70701	22.88684
E3-2	9.01106	18.50296

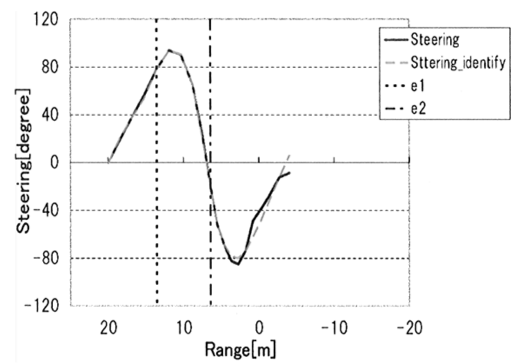


Fig. 19. Comparison of measured and reproduced steering profiles (E1-1, three modes).

- *Mode D:* The magnitude of all coefficients is small in this mode. This implies that the control effort was less required compared with other mode. The negative d_2 was used by the E2.

As discussed above, although the control scenario varies from driver to driver, the list of coefficients can be used to analyze the drivers' characteristics in a quantitative manner.

Note that the parameters of the E1 may show different tendency (especially in the Mode B) when the braking operation is included in the model explicitly by using the strategy for the multioutput case described in Section V-B. Although this point seems interesting, the analyses emphasizing the steering can be a good first step to understanding the case of the multioutput model. This point will be addressed in our future work.

2) Parameters in Switching Conditions (e_i): Although all three drivers seem to switch the control law appropriately, the variance of the switching condition of all trials (twelve trials) shows the interesting phenomena. The average and variance of each switching point e_i are listed in Table V, wherein the variance of the second switching point e_2 takes the smallest value. This is because the second switching is caused by the change of the gazing point of the driver (from back of the truck to the far front of the road; see Fig. 11), and this change always occurs at almost the same geometrical situation between vehicles. On the other hand, the variance of the third switching point e_3 takes the extremely largest value. This implies that the third switching has no clear reasoning from the viewpoint of decision making since the driver can find no clear target during Modes C and D. These analyses represent the inherent advantage of the modeling based on the controller switching model.

3) Number of Modes: We have investigated the case where the number of modes is reduced to three. First of all, the perfor-

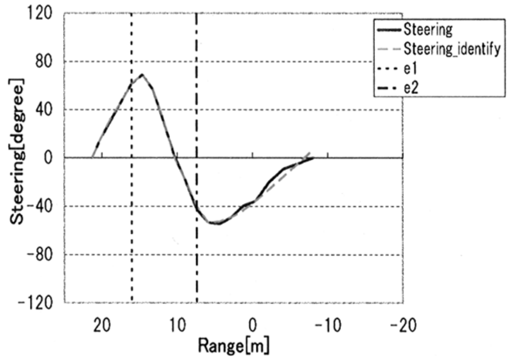


Fig. 20. Comparison of measured and reproduced steering profiles (E1-2, three modes).

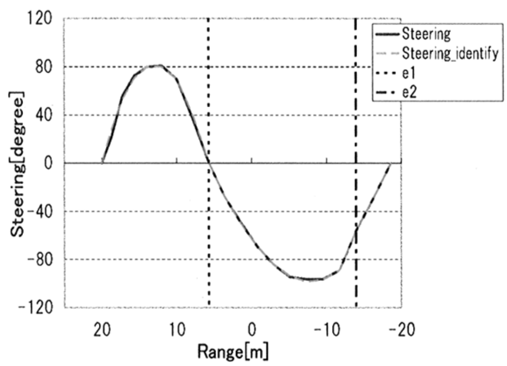


Fig. 21. Comparison of measured and reproduced steering profiles (E2-1, three modes).

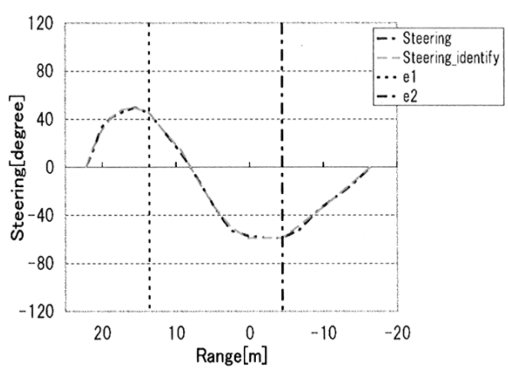


Fig. 22. Comparison of measured and reproduced steering profiles (E2-2, three modes).

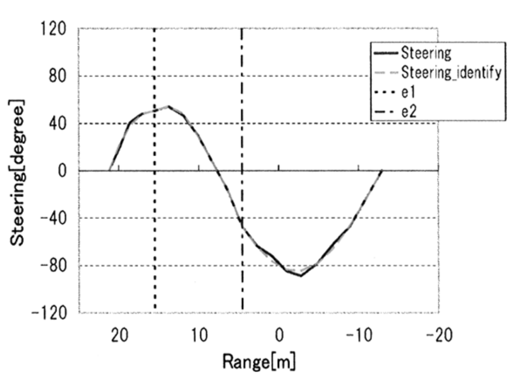


Fig. 23. Comparison of measured and reproduced steering profiles (E3-1, three modes).

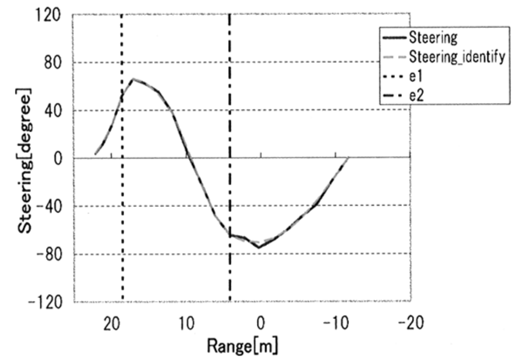


Fig. 24. Comparison of measured and reproduced steering profiles (E3-2, three modes).

mance index J is listed in Table VI, wherein the modeling accuracy was degraded by decreasing the number of modes. In addition, the identification results are shown in Figs. 19–24. As we can see in these figures, the switching point e_3 in the four-mode model tends to disappear in the three-mode model [except for Fig. 21 (E2-1)]. These results back up the discussion as for the unclear switching, i.e., the largest variance of e_3.

VI. Conclusions

In this paper, we have developed the modeling strategy of the human driving behavior based on the expression as PWP model, especially focusing on the driver's collision avoidance maneuvers. The driving data was collected by using the driving simulator, which provides 3-D stereoscopic immersive virtual environment. The identification problem of the PWP model was formulated as the MILP by transforming the logical switching conditions in the PWP model into inequalities. By applying our proposed modeling framework, it has been found that the driver appropriately switches the "control law" according to the sensory information. In addition, we have compared the driving characteristics of the beginner driver and the expert driver. As for the results, we could see that the beginner driver tends to signify the range information rather than the lateral displacement between cars in the mode of small range (Mode B). Moreover, the comparison between the four-mode model and the three-mode model has been made and discussed, related to the variance of the switching points. These quantitative discussions enable us to interpret not only the physical (operational) meaning of the driving skill but the decision-making aspects (switching conditions) of the the driving behavior as well. The analysis in more complicated situation, and the application of the obtained results to the design of the collision avoidance support system, are the subjects of future work.

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